KOMPLEX ANALYS VID VÄRLDENS ÄNDE

Här kommer nu det sista informationsbrevet inför vårt möte i Trosa. Tid och plats är alltså fredagen den 14 mars kl. 14.00 till söndagen den 16 mars kl. 14.00 på Trosa Stadshotell, Västra Långgatan 19, Trosa (telefon +46-(0)156-17070).

Ni som kommer med bil tar av från E4:an vid Vagnhärad och följer i Trosa skyltarna mot stadshotellet. Man kan också åka med buss 702 från Södertälje Syd (där SJ-tågen stannar) eller från Södertälje Hamn (dit pendeltågen från Stockholms centralstation går). Bussen avgår från Södertälje Hamn 10.45 och 12.15 (samt även 13.45) och fem minuter senare från Södertälje Syd. Det pendeltåg man måste ta från centralstationen för att hinna till Södertälje Hamn innan 12.15 avgår 11.15.

PROGRAM

FREDAG

14.00 Ankomst, kaffe
15.00 Information
15.15–16.00 Christer Kiselman
16.15–17.00 Stefan Halvarsson
17.15–18.00 Burglind Juhl-Jöricke
19.00 Middag

LÖRDAG

08.00–10.00 Frukost
10.30–11.30 Lars Hörmander
12.00 Lunch
14.30 Kaffe
15.15–16.00 Ulf Backlund
16.15–17.00 Jörgen Boo
17.15–18.00 Finnur Lárusson
19.00 Middag

SÖNDAG

08.00–10.00 Frukost
10.15–11.00 Gennadi Henkin
11.15–12.00 Bo Berndtsson
12.00 Lunch
14.00 Avresa

DELTAGARE OCH RUMSFÖRDELNING

101 Timur Sadykov (Krasnojarsk) & August Tsikh (Krasnojarsk)
102 Mats Andersson (Göteborg)
103 Burglind Juhl-Jöricke (Berlin)
104 Hasse Carlsson (Göteborg)
105 Bo Berndtsson (Göteborg)
106 Jan-Erik Björk (Stockholm)
109 Andreas Nilsson (Stockholm) & Anders Olofsson (Stockholm)
110 Peter Ebenfelt (Stockholm) & Mikael Passare (Stockholm)
112 Nils Øvrelid (Oslo)
115 Ozan Öktem (Stockholm)
116 Lars Alexandersson (Göteborg) & Jörgen Boo (Göteborg)
118 Jim Arlebrink (Borås) & Henrik Delin (Göteborg)
119 Peter Grenholm (Uppsala) & Björn Ivarsson (Uppsala)
122 Ulf Backlund (Umeå) & Anders Fällström (Umeå)
123 Finnur Lárusson (London) & Ragnar Sigurdsson (Reykjavik)
202 Gennadi Henkin (Paris)
203 Mikael Forsberg (Stockholm)
204 Jan Boman (Stockholm)
208 Lars Hörmander (Lund)
212 Alain Yger (Bordeaux)
215 Wang Xiaoqin (Gävle)
216 Leif Abrahamsson (Uppsala) & Stefan Halvarsson (Uppsala)
217 Johan Thorbiörnson (Sundsvall) & Yang Xing (Umeå)
218 Nikolay Shcherbina (Göteborg)
219 Per Manne (Bergen)
221 Jockum Aniansson (Stockholm) & Lars Filipsson (Stockholm)
222 Erik Løw (Oslo)
223 Christer Kiselman (Uppsala)

TITLAR OCH SAMMANFATTNINGAR

Ulf Backlund: The Gleason property for Reinhardt domains

Let $A(\Omega)$ be the uniform algebra consisting of the functions which are holomorphic on a domain Ω in $\mathbb{C}^n$ and continuous on the closure of Ω. A bounded domain Ω in $\mathbb{C}^n$ is said to have the Gleason A-property at $p \in \Omega$ if the maximal ideal in $A(\Omega)$ consisting of functions vanishing at p is algebraically generated by the coordinate functions $z_1 - p_1, \ldots, z_n - p_n$. If Ω has the Gleason A-property at every point $p \in \Omega$, then Ω is said to have the Gleason A-property.

We prove that bounded pseudoconvex complete Reinhardt domains in $\mathbb{C}^2$ with C^2-boundary have the Gleason A-property.

Bo Berndtsson: Some variations on Hörmander's L^2-estimates for $\bar\partial$

The proof of Hörmander's L^2-estimates for solutions to the $\bar\partial$-equation is based on an L^2-identity for differential forms. I will discuss a differential identity, which implies the L^2-identity, and survey some applications of this formula to various $\bar\partial$-estimates.

Jörgen Boo: Canonical homotopy operators for $\bar\partial$

Let $D = \{\rho < 0\}$ be a strictly pseudoconvex domain in $\mathbb{C}^n$. The spaces L^2_α of (p,q)-forms, defined with respect to the metric induced by $\Omega = (-\rho)\, i\partial\bar\partial \log(1/-\rho)$ and with weights $(-\rho)^\alpha$, are natural for several reasons:

(1) The metric reflects the well known difference in the behaviour of $\bar\partial$ in normal and tangential directions near the boundary.

(2) Some explicit solution operators K_α for $\bar\partial$ turn out to be canonical with respect to L^2_α (in the ball; in the general case the formulas merely are approximately canonical).

(3) When α tends to zero, the canonical operators K_α tends to the canonical solution operator K_b for $\bar\partial_b$ on ∂D. Moreover, one can represent K_α in terms of the boundary values of $K_{\alpha-1}$ in the domain $\{\rho(z) + |w| < 0\} \subset \mathbb{C}^{n+1}$.

Stefan Halvarsson: On generalized indicator functions

We introduce a new measure of growth which can be used as a refinement of the relative order for convex and entire functions. The indicator of growth functions from the classical theory can be generalised to any relative order and we prove a generalised (circular) indicator of growth theorem.

Gennadi Henkin: On boundaries of complex analytic varieties

Let M be a compact, oriented, odd-dimensional, smooth submanifold of a complex manifold X. Several interesting problems of complex analysis, geometry and mathematical physics can be reduced to the following fundamental question: when is M the boundary of a complex analytic variety or more generally of a holomorphic chain T in X, i.e. when is $M = bT$, where T is holomorphic chain?

For the case $dim\, M \geq 3$ and X is Stein manifold a remarkable solution of this problem is given by a result of Harvey-Lawson (1974): $M = bT$ iff M is a maximally complex CR-manifold, i.e. iff the tangent spaces of M contain a complex subspace of maximal possible dimension.

If X is not Stein the condition of maximal complexity for M is obviously necessary but generally not sufficient for the conclusion $X = bT$.

For the case $M \subset X = \mathbb{C}P^n$, or more generally M belongs to projective manifold, the problem (posed by J.King (1978)) can be solved in the following manner (Dolbeault-Henkin (1996)): $M = bT$ iff for any linear subspace $\mathbb{C}P^k_\xi \subset \mathbb{C}P^n$, $k = n - \frac{1}{2}(dim_{\mathbb{R}} M - 1)$ the intersection $M \cap \mathbb{C}P^k = \gamma$ is a closed (not necessarily connected and sometimes empty) curve with the property $\gamma = bS$, where S is a holomorphic 1-chain (Riemann surface with possible singularities and multiplicities).

The lecture will contain a review of several recent results on the formulated problems, applications and conjectures, in particular:

- Dinh (1996)- version of Wermer-Alexander-Harvey-Lawson theorem for rectifiable CR-manifolds (Alexander-King problem)
- Dolbeault-Henkin and Sarkis (1997) generalisation of the Hartogs-Levi theorem for CR-meromorphic functions (Harvey-Lawson problem)
- Fabre (1996) examples of strictly pseudoconvex domains in $\mathbb{C}P^2$ intersecting all algebraic curves in $\mathbb{C}P^2$ (Hirai problem)
- Henkin-Passare (1996) generalisation of Lie-Griffiths inverse Abel theorem
- Conjectures and partial results on inverse Newton-Ivory-Arnold theorem.

Lars Hörmander: Spacelike asymptotics of positive energy solutions of the Klein-Gordon equation

For a temperate solution of the Klein-Gordon equation in $\mathbf{R}^{1+n}$,

$$\Box u + u = 0; \quad \Box = \partial_t^2 - \Delta, \quad \Delta = \sum_1^n \partial_j^2,\ \partial_j = \partial/\partial x_j, \tag{1}$$

the Fourier transform is supported by the hyperboloid $\{(\tau,\xi) \in \mathbf{R}^{1+n}; \tau^2 = |\xi|^2 + 1\}$. It is said to be a positive energy solution if it is supported by the sheet where $\tau > 0$. The purpose of the lecture is to discuss how fast such solutions can decay in spacelike directions. For the Cauchy data

$$u = u_0,\ \partial_t u = u_1 \quad \text{when } t = 0, \tag{2}$$

positivity means that $\hat{u}_1(\xi) = -i\langle\xi\rangle\hat{u}_0(\xi)$ when $\xi \in \mathbf{R}^n$, where $\langle\xi\rangle = (1 + \langle\xi,\xi\rangle)^{\frac{1}{2}}$. Using the branching of the square root when ξ becomes a purely imaginary unit vector it is easy to see that there are no positive energy solutions with Cauchy data $O(e^{-a|x|})$ at infinity if $a > 1$. More precisely, we prove that there are no positive energy solutions $\neq 0$ with

$$\int_{\mathbf{R}^n} (|u_0(x)| + |u_1(x)|) \exp(|x| + c\sqrt{|x|})\, dx < \infty, \quad x \in \mathbf{R}^n, \tag{3}$$

where c is a positive constant, but for every $a \in (0, \frac{1}{2})$ there exist such solutions with

$$(|u_0(x)| + |u_1(x)|) \exp(|x| + |x|^a) \leq C, \quad x \in \mathbf{R}^n. \tag{4}$$

There are stronger results of Denjoy-Carleman type.

If $u^{\pm}$ is the positive (negative) energy component of a solution of the Klein-Gordon equation with Cauchy data of compact support, then

$$u^{\pm}(t, r\omega) = \pm\frac{1}{2}i(2\pi r)^{-n/2}e^{-r}\Big((-1)^{\nu_0}(2\nu_0+1)!!r^{-1-\nu_0}t\big(\hat{v}_0(i\omega)+O(r^{-1})\big) + (-1)^{\nu_1}(2\nu_1-1)!!r^{-\nu_1}\big(\hat{v}_1(i\omega)+O(r^{-1})\big)\Big), \quad r>0,\ |\omega|=1, \tag{5}$$

where $\hat{v}_j$ is the Fourier-Laplace transform of u_j with factors $1+\langle\zeta,\zeta\rangle$ removed, so $e^{|x|}u^{\pm}(t,x)$ can only decay as a power of $1/|x|$ in generic directions.

The point of the Klein-Glordon equation is its relativistic invariance. From that point of view a positive energy solution cannot decay with the speed (4): we prove that if u is a positive energy solution with

$$|u(t,x)| \le C(|x|^2-t^2)^{-\frac{1}{4}n-\frac{1}{2}\gamma}e^{-\sqrt{|x|^2-t^2}}, \text{ if } t^2<\delta|x|^2,$$

where $\gamma>0, \delta>0$, then $u=0$. The proof depends on an analytic continuation inside the complex hyperboloid

$$\mathcal{H}_{\mathbf{C}} = \{\zeta\in\mathbf{C}^n; 1+\langle\zeta,\zeta\rangle=0\},$$

and an application of the edge of the wedge theorem at the wedge formed by purely imaginary elements, which is a hyperboloid with one sheet.

Burglind Juhl-Jöricke: Local polynomial hulls of discs near isolated parabolic points

Bishop was the first to understand that 2-speres imbedded into $\mathbb{C}^2$ must have complex tangencies. He classified complex tangencies of 2-surfaces imbedded into strictly pseudoconvex boundaries: this are elliptic, hyperbolic and parabolic points. 2-surfaces are never locally polynomially convex near elliptic points (Bishop), but always near hyperbolic points (Stout). Parabolic points arise generically for families of 2-sheres, for example in case of collapsing of an elliptic point and a hyperbolic one. It was open whether $\mathbb{C}^2$ discs are always locally polynomially convex near isolated parabolic points of index zero. The answer is no, moreover, it is shown that if the essential local hull is not empty then its trace on the disc is one "petal". We call a petal the closure K of a domain, contained in a cone with vertex the parabolic point, such that K is the union of the boundaries of analytic discs, every boundary containing the parabolic point.

Christer Kiselman: The Behnke–Peschl condition is sufficient for weak lineal convexity

In an article published in the *Mathematische Annalen* in 1935, Heinrich Behnke and Ernst Peschl introduced a notion of convexity called *Planarkonvexität*, nowadays known as *weak lineal convexity*. They showed that for domains in the space of two complex variables with boundary of class C^2, this property implies that a differential inequality is satisfied at every boundary point. We shall prove that, conversely, the differential inequality is sufficient for weak lineal convexity.

Finnur Lárusson: New constructions of pluricomplex functions on manifolds

This is a report on joint work with Ragnar Sigurdsson. Let X be a complex manifold and $\mathcal{A}$ be the family of holomorphic maps from the closed unit disc into X. A disc functional on X is a map $H : \mathcal{A} \to [-\infty, \infty)$. The envelope of H is the function $EH : X \to [-\infty, \infty)$ such that $EH(x)$ is the infimum of the numbers $H(f)$ over $f \in \mathcal{A}$ with $f(0) = x$. Through work of Poletsky, it has transpired that the envelopes of certain disc functionals on domains in $\mathbb{C}^n$ are plurisubharmonic. We discuss a disc functional H, whose envelope, when plurisubharmonic, is a generalized pluricomplex Green function, which can be characterized as the largest negative plurisubharmonic function on X having Lelong numbers no smaller than a given non-negative function on X. We have shown that with some restrictions, EH is plurisubharmonic when X belongs to a large class of manifolds, containing for instance all domains in Stein manifolds, all Riemann surfaces, and all covering spaces of projective algebraic manifolds.