

# NORDAN 2001 - OSLO MAY 4-6 - PROGRAM

## Friday May 4th

·1545:Welcome/Info

·1600-1645:Mikael Passare, Stockholm - "Amoebas, Monge-Ampère measures, and triangulations of the Newton polytope"

·1645-1715:Coffee/cake

·1715-1800:Henrik Laurberg Pedersen, København - "Pick functions related to entire functions having negative zeros"

·1815-1900:Niklas Lindholm, Göteborg - "Sampling in complex analysis"

·2000:Dinner

19:30

## Saturday May 5th

·0900-0945:Mats Andersson, Göteborg - "Fundamental solutions to  $\bar{\partial}$  in  $\mathbb{C}^n$  and representation formulas with weights"

·1000-1045:Alexander Ulanovskii, Stavanger - "On certain quasi-analytical properties of convolutions of measures"

·1100-1145:John Erik Fornæss, Ann Arbor - "Attractors in complex dynamics."

·1230:Lunch

·1530-1615:Finnur Larusson, London,Ontario - "Abstract homotopy theory and Gromov's Oka principle"

·1615-1645:Coffee/cake

·1645-1730:Frank Kutzschebauch, Uppsala - "Embeddings and compact group actions on  $\mathbb{C}^n$ "

·1745-1830:Frank Wikstrøm, Umeå - "Pluricomplex Green functions with multiple poles"

·2000:Dinner

## Sunday May 6th

·0900-0945:Gerd Schmalz,Bonn - "Simultaneous linearization of the isotropic automorphisms of a CR-manifold"

·1000-1045:Christophe Mourougane,Paris - "Interpolation in non-positively curved Kähler manifolds"

·1100-1145:Bjørn Ivarsson, Uppsala - "On the behaviour of strictly plurisubharmonic functions near real hypersurfaces"

·1145-1230:Hotel checkout

·1230:Lunch

## NORDAN 2001

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**Mats Andersson** *Fundamental solutions to  $\bar{\partial}$  in  $\mathbf{C}^n$  and representation formulas with weights.*

*Abstract.* Let  $\delta_{z-a}$  denote contraction with the vector field

$$2\pi i \sum_1^n (z_j - a_j) \frac{\partial}{\partial z_j}$$

and consider a current solution  $u = u_{n,n-1} + \cdots + u_{1,0}$  ( $u_{k,k-1}$  being a  $(k, k-1)$ -current) to

$$(1) \quad (\delta_{z-a} - \bar{\partial})u = 1 - [a],$$

where  $[a]$  denote the  $(n, n)$ -current point evaluation at  $a \in \mathbf{C}^n$ . In one variable the only possibility is the Cauchy kernel

$$u(z) = \frac{1}{2\pi i} \frac{dz}{z-a},$$

but in higher dimensions there are many choices. Any solution to (1) gives rise to a generalized Cauchy-Fantappie-Leray formula. We will describe a quite general way to obtain representation formulas with weight factors from solutions to (1). By allowing a more general singularity than just a point a similar approach yields weighted formulas for  $\bar{\partial}$  on complex manifolds.

One can define Fourier transforms of forms and currents, and it turns out that the Fourier transform of a solution to (1) with  $a = 0$  again is a solution to essentially the same equation. We will indicate applications to non-holomorphic operator calculus.

**John Erik Fornæss** *Attractors in complex dynamics.*

*Abstract.* Attractors play an important role in dynamics. The most basic ones are attracting fixed points (or periodic points). There has also been a lot of investigation about attractors that are extended in space. In that case, the focus has been on the investigation of detailed dynamical properties inside the attractor. Examples are Henon attractors and Lorentz attractors. One can also study attractors that are extended in space, but ignoring the internal dynamics. The main object then is the basin of the attractor. This is joint work with Bonifant.

**Bjørn Ivarsson** *On the behaviour of strictly plurisubharmonic functions near real hypersurfaces.*

*Abstract.* For a smooth plurisubharmonic function  $u$  let  $Mu$  be the product of the eigenvalues of the complex Hessian. Assume that  $S$  is a smooth real hypersurface in  $\mathbf{C}^n$  such that its Levi form is positive semidefinite. We show that if there is a smooth plurisubharmonic function  $u$  defined on a one-sided neighborhood  $U$  of  $S$  such that  $Mu \geq 1$ ,  $\lim_{z \rightarrow z_0 \in S} u(z) = 0$  and  $u$  is Lipschitz on the closure of  $U$ , then the Levi form of  $S$  is positive definite.

**Frank Kutzschebauch** *Embeddings and compact group actions on  $\mathbf{C}^n$ .*

*Abstract.* We will consider proper holomorphic embeddings  $\varphi : \mathbf{C}^k \hookrightarrow \mathbf{C}^n$  ( $0 < k < n$ ) up to equivalence by holomorphic automorphisms (orbits with respect to the natural action of  $Aut(\mathbf{C}^n) \times Aut(\mathbf{C}^k)$ ). Also we will consider holomorphic actions of compact Lie groups on  $\mathbf{C}^n$  up to equivalence given by conjugation inside the group of holomorphic automorphisms  $Aut(\mathbf{C}^n)$ . The main point of the talk is to explain how to pass from non-equivalent embeddings to non-equivalent actions (the method found by H. Derksen and myself to solve the **Holomorphic Linearization Problem**).

**Finnur Larusson** *Abstract homotopy theory and Gromov's Oka principle*

*Abstract.* Abstract homotopy theory takes place in a category satisfying a list of axioms due to Quillen, giving a reasonable notion of two arrows being homotopic. This encompasses ordinary homotopy theory of topological spaces and simplicial sets, homological algebra, and much more. Recent applications in arithmetic geometry have attracted much attention.

The Oka Principle is an important theme in complex analysis with a long history. Roughly speaking, it states that on a complex submanifold of Euclidean space, analytic problems of a cohomological nature have only topological obstructions. A famous theorem of Gromov is an instance of this, giving sufficient conditions for a continuous map between two complex manifolds to be homotopic to a holomorphic map.

The talk will give a brief overview of these two topics and describe how Gromov's Oka Principle can be placed in an abstract homotopy-theoretic context and interpreted in purely holomorphic terms, without reference to continuous maps. To this end, we embed the category of complex manifolds into a Quillen category, where we can then do homotopy theory with them. The conclusion of Gromov's theorem turns out to be equivalent to a property called excision, which is familiar from topology and appears nowadays in algebraic geometry.

**Niklas Lindholm** *Sampling in complex analysis.*

*Abstract.* We will discuss sampling in one and several complex variables, give a short survey of results and compare some different cases.

**Christophe Mourougane** *Interpolation in non-positively curved Kähler manifolds.*

*Abstract.* I will explain how to extend to any simply connected Kähler manifold with non-positive sectional curvature some conditions for interpolation in  $\mathbf{C}$  and in the unit

disk given by Berndtsson, Ortega-Cerdà and Seip. The main tools are  $L^2$  estimates and a comparison theorem for the Hessian in Kähler geometry due to Greene, Wu and Siu, Yau.

**Mikael Passare** *Amoebas, Monge-Ampère measures, and triangulations of the Newton polytope.*

*Abstract.* The amoeba of a holomorphic function  $f$  is by definition the image in  $\mathbf{R}^n$  of its zero locus under the simple mapping  $(z_1, \dots, z_n) \mapsto (\log |z_1|, \dots, \log |z_n|)$ . The terminology was introduced in the nineties by the famous (biologist and) mathematician Israel Gelfand and his co-authors. In this talk we shall discuss a natural convex potential function  $N_f$ , with the property that its Monge–Ampère mass is concentrated to the amoeba. We obtain results of two kinds; by approximating  $N_f$  with a piece-wise linear function we get striking combinatorial information regarding the amoeba and the Newton polytope of  $f$ ; by computing the Monge–Ampère measure we find sharp bounds for the area of amoebas in  $\mathbf{R}^2$ . All this is joint work with Hans Rullgård.

**Henrik Laurberg Pedersen** *Pick functions related to entire functions having negative zeros.*

*Abstract.* At the Fifth International Symposium on Orthogonal Polynomials, Special Functions and Applications (held in Patras, Greece 1999) Dimitar Dimitrov conjectured that the function

$$f(x) = \frac{\log \Gamma(x+1)}{x \log x}, \quad x > 0$$

has a completely monotone derivative, i.e. satisfies the inequalities  $f^{(2k-1)}(x) > 0$  and  $f^{(2k)}(x) < 0$  for  $x > 0$  and for all  $k \geq 1$ . It had earlier been proved that  $f$  increases and is concave. The function  $f$  appears in the study of the asymptotic behaviour of the Lebesgue measure of the  $n$ -dimensional unit ball. I shall briefly describe how this conjecture was affirmed (joint work with Christian Berg, University of Copenhagen) and I shall present a generalization of the result to a certain class of entire functions having negative zeros. The proofs use methods from complex analysis: it turns out that a suitable holomorphic extension of the function  $f$  (or any function of a certain class) is a Pick function (i.e. has non-negative imaginary part in the upper half-plane). Any Pick function has an integral representation. From the corresponding representation one obtains as a corollary the complete monotonicity of the derivative.

**Gerd Schmalz** *Simultaneous linearization of the isotropic automorphisms of a CR-manifold.*

*Abstract.* An automorphism with fixed point  $p$  of a real-analytic CR-manifold  $M$  with non-degenerate Levi form is uniquely determined by its first and second order derivatives at  $p$ . Belošapka and Loboda proved that the isotropic automorphisms of a non-quadratic hypersurface  $M$  are determined by a part of its first order derivatives at  $p$ . For strictly pseudoconvex hypersurfaces one can find normal coordinates where all isotropic automorphisms are linear mappings (Kružilin and Loboda). For CR-manifolds of higher codimension  $k > 1$  the following results are known: For hyperbolic, non-semiquadratic and

elliptic, non-exceptional manifolds of codimension 2 in  $\mathbf{C}^4$  the isotropic automorphisms can be simultaneously linearized (Schmalz, Schmalz and Ežov). The same is true for rigid CR-manifolds (general position for  $3 \leq k \leq n^2 - 3$  with  $n = \text{CR-dim } M$ ).

**Alexander Ulanovskii** *On certain quasi-analytical properties of convolutions of measures.*

*Abstract.* The aim of the talk is to present two different non-classical properties of quasi-analyticity. We shall also discuss connections between these properties and some problems and results in Complex and Harmonic Analysis.

**Frank Wikstrøm** *Pluricomplex Green functions with multiple poles.*

*Abstract.* The pluricomplex Green function with multiple poles was introduced by Lelong in 1989, but did not receive much attention until a few years ago. I will talk about some properties of this function and tell you something about what is known and what is still open. One example of a topic I will cover is the following. Let  $A = \nu_1 w_1 + \dots + \nu_p w_p$  be a pole set in the domain  $\Omega$ . It is straight-forward to check

that

$$\sum_j \nu_j g(z; w_j) \leq g(z; A) \leq \min_j \nu_j g(z; w_j).$$

We will look at descriptions of the sets where these inequalities are equalities.

- Lars Alexandersson , Linköping
- Bo Berndtsson , Göteborg
- Jan Boman , Stockholm
- Jörgen Boo , Sundsvall
- Stefan Borell , Sundsvall
- Hasse Carlsson , Göteborg
- Linus P. Carlsson , Umeå
- Urban Cegrell , Umeå
- Anders Fällström , Umeå
- Lars Filipsson , Västerås
- Lars Hörmander , Lund
- Robert Juhlin , Stockholm
- Burglind Juhl-Jöricke , Uppsala
- Christoph Lampert , Göteborg
- Oscar Lemmers , Amsterdam
- Sam Lodin , Sundsvall
- Erik Løw , Oslo
- Jon Ingolfur Magnusson , Reykjavik
- Per Erik Manne , Bergen
- Klas Markström , Umeå
- Evgeny Materov , Tübingen
- Henrik Petersson , Växjö
- Alexander Rashkovskii , Stavanger
- Maria Roginskaya , Göteborg
- Bjarte Rom , Trondheim
- Hans Rullgård , Stockholm
- Timur Sadykov , Stockholm
- Sebastian Sandberg , Göteborg
- Kristian Seip , Trondheim
- Nikolay Shcherbina , Göteborg
- Ragnar Sigurdsson , Reykjavik
- August Tsikh , Krasnojarsk
- Yang Xing , Umeå
- Nils Øvrelid , Oslo
- Per Åhag , Sundsvall