NORDAN ÅTTA

Nösund, 14-16 maj 2004

PROGRAM

Fredag 14 maj

13.00 Lunch

15.00 - 15.45 John Wermer

16.00 - 16.45 Hans Rullgård

19.30 Middag

Lördag 15 maj

7.00 - 9.00 Frukost

9.15 - 10.00 Anders Fällström

10.15 - 11.00 Stephanie Nivoche

11.15 - 12.00 Josip Globevnik

12.00 Lunch

14.00 - 14.45 Alain Yger

15.00 - 15.45 Jan-Erik Björk

17.00 Bad

19.30 Middag

Söndag 16 maj

7.30 - 9.00 Frukost

9.15 - 10.00 Berit Stensones

10.15 - 11.00 Alexandre Sukhov

11.15 - 12.00 Jan Wiegerinck

12.00 Lunch

TITLAR OCH SAMMANFATTNINGAR

John Wermer

Rational approximation on the unit sphere in $\mathbb{C}^2$

For X a compact set in $\mathbb{C}^n$, $R(X)$ denotes the uniform closure on X of rational functions on $\mathbb{C}^n$ which are regular on X. When does $R(X) = C(X)$? A necessary condition is that X be rationally convex. In 1972 Richard Basener gave an example of a rationally convex set X on the unit sphere S in $\mathbb{C}^2$ with $R(X)$ distinct from $C(X)$. We shall discuss the question when equality holds for certain classes of subsets of S.

Hans Rullgård

Amoebas in complex geometry

Consider a Laurent polynomial in n complex variables

$$f(z) = \sum_{\alpha \in A} a_\alpha z^\alpha$$

where A is a finite subset of $\mathbf{Z}^n$ and $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$. The *Newton polytope* of f is defined as the convex hull P_f of A in $\mathbf{R}^n$. The *amoeba* of f is defined to be the set

$$\mathcal{A}_f = \operatorname{Log}(\{z \in (\mathbf{C} \setminus 0)^n; f(z) = 0\}) \subset \mathbf{R}^n$$

where

$$\operatorname{Log}(z_1, \ldots, z_n) = (\log |z_1|, \ldots, \log |z_n|).$$

Every connected component of $\mathbf{R}^n \setminus \mathcal{A}_f$ is an open convex set, and the set of these connected components is in one-to-one correspondense with the convergent Laurent series expansions of $1/f$. Moreover, the set of complement components is in a one-to-one correspondence with a subset of $P_f \cap \mathbf{Z}^n$.

The amoeba can be approximated by a piecewise linear set $\mathcal{S}_f$, called the *spine* of the amoeba. The spine is a *tropical variety* defined by a *tropical polynomial*

$$g(x) = \max_{\alpha \in A'}(c_\alpha + \langle \alpha, x \rangle)$$

where A' is a subset of $P_f \cap \mathbf{Z}^n$. In certain cases, the coefficients c_α can be computed by the series expansion

$$c_\alpha = \log |a_\alpha| + \operatorname{Re} \sum_{k \in K_\alpha} \frac{(-k_\alpha - 1)!}{\prod_{\beta \neq \alpha} k_\beta!} (-1)^{k_\alpha - 1} \prod_{\beta \in A} a_\beta^{k_\beta},$$

Josip Globevnik

Analyticity on circles

Let U be the union of a family S of circles in the complex plane and let f be a continuous function on U which extends holomorphically from each circle C in S to the disc bounded by C. When does it follow that f is holomorphic in the interior of U? We will show how some tools from several complex variables can be used to deal with this problem.

Alain Yger

Multidimensional residue currents as a tool for intersection or division theory ?

Residues associated to regular sequences are defined in the algeraic context in terms of traces; nevertheless (as it happens for example in J. Lipman's approach), the residue corresponding to a regular sequence $(f_1, ..., f_p)$ is not defined alone, but together with the whole list of residues attached to the sequences $(f_1^{k_1+1}, ..., f_p^{k_p+1})$, $k \in \mathbb{N}^p$. Moreover, it seems that the key results linked with multidimensional residue calculus involve the behavior of such a sequence, as for example the Briançon-Skoda theorem in its generalized version ($\overline{I^{n+p-1}} \subset I^p$ for $p \in \mathbb{N}^*$ and I some ideal in a regular local ring). Also the fact that the Coleff-Herrera current $\overline{\partial}\frac{1}{f}$ is annihilated by the ideal (in the complete intersection case) seems formally related to the fact that the sequence $f^{\underline{k}}\overline{\partial}\frac{1}{f^{\underline{k}+1}}$ does not depend of $\underline{k} \in \mathbb{N}^p$! In this talk, we will focuse on such questions, indicate different technics to "multiply" residual currents in a robust way (thanks to $\mathcal{D}$-module ideas introduced by J. E. Björk), in relation with some work in progress with August Tsikh, and analyse potential consequences of such constructions towards problems in intersection theory (precise estimates of Lojasiewicz exponents at infinity, search for the realisation in terms of multiplication of residue currents of the multi-index of contact introduced by P. Tworzewski in improper analytic intersection theory). As it appears in the relation beween residue calculus and trace theory (for example in the formulation of Abel's theorem about the meromorphy of the trace and its inverse), it seems interesting to analyse how residue currents depend on additional parameters in order to derive some way to control either intersection or division problems in commutative algebra. Expanded notes of the talk will be prepared to be available for Nordan.

Berit Stensones

The Michael Problem revisited

The Michael problem asks if one can find a sequence of entire maps F_j from $\mathbb{C}^n$ to $\mathbb{C}^n$ for some n such that if D_j is the image of $\mathbb{C}^n$ under F_1 composed with F_2 composed with . . . F_j such that the intersection between the sets D_j is empty. We shall give a positive answer to this question and also focuse on part of the proof that involves building and solving certain d-bar problems.

Alexandre Sukhov

Some aspects of analysis on almost complex manifolds

We prove an analog of Fefferman's mapping theorem for almost complex manifolds of complex dimension 2. We show that a biholomorphism between two smoothly bounded strictly pseudoconvex domains in such manifolds extends as a smooth diffeomorphism between their closures. Our proof relates on some techniques which are of independent interest for analysis on almost complex manifolds: estimates of the Kobayashi-Royden pseudometric, geometry of pseudoholomorphic discs attached to real submanifolds, the reflection principle. One of the main tools of our approach is a version of the scaling method adapted to the almost complex case. We hope that it will be useful in the study of other questions related to the almost complex analysis.

Jan Wiegerinck

The pluripolar hull of a graph

A set $E \subset \mathbb{C}^n$ is called pluripolar if it is contained in the $-\infty$ set of a plurisubharmonic function defined on a neighborhood of E. One may wonder when such an E is complete, i.e. *equal* to the $-\infty$ set of a PSH-function. The graph Γ_f of a holomorphic functions f on a domain in $\mathbb{C}$ is a typical example of a pluripolar subset of $\mathbb{C}^2$. This graph is complete in a domain $\Omega \subset \mathbb{C}^2$ if it is equal to its pluripolar hull in Ω:

$$E^*_\Omega = \{z \in \Omega : h|_E = -\infty \text{ implies } h(z) = -\infty,\ h \in \mathrm{PSH}(\Omega)\}.$$

In this talk I will report on joint work with Armen Edigarian dealing with the following results. Even if f cannot be extended over the boundary of D, the graph Γ_f of f may have a non-trivial pluripolar hull, (and then it is not complete). When the domain D of f is the complement of a polar subset of a larger domain $\tilde{D}$, we can give a precise description of $E^*_{\tilde{D}}$. It will be the graph of an extension of f to a set X, $D \subset X \subset \tilde{D}$. A condition in terms of classical potential theoretic properties of f determines which points belong to $X \setminus D$.