

NORDAN 2011

May 6–May 8, Röstånga



Vetenskapsrådet



LUNDS
UNIVERSITET

Nordan 2011

Welcome to the 15th annual Nordic Symposium in Complex Analysis. This year, Nordan pushes the southern frontiers of its dominion. For the first time ever, the conference is organized by Lund University and takes place in Röstånga, right in the middle of Skåne, the southernmost district in Sweden. One might even say that Nordan for the first time ever is held in Denmark, at least from a historical point of view. It wasn't until 1658 that Skåne became part of Sweden.



Lund University, main building

Lund university, or *Regia Academia Carolina* as it was originally known, was founded in 1666, foremost as an attempt to speed up the "swedification" of the new territories in the outskirts of the country. Lund University is not, contrary to popular belief, the second oldest Swedish university – the university in Dorpat (present day Tartu) was founded in 1632, and the university in Åbo was founded in 1640.

Even older than Lund University is Röstånga Gästgivaregård. The inn was mentioned in official records as early as 1647. The inn was originally located elsewhere, but was moved to its current location in the mid 18th century. Unfortunately, the inn was destroyed in a fire in 1906 and was rebuilt and reopened in 1909.

We would like to thank the Swedish Research Council as well as the Center for Mathematical Sciences, Lund University for their generous financial support.

Program

Friday

- 14.30 – 15.15 Bo Berndtsson, Göteborg:
A Brunn-Minkowski theorem for Fano manifolds and uniqueness for Kähler-Einstein equations
- 15.15 – 15.45 Coffee break
- 15.45 – 16.30 Jasna Przelj, Ljubljana:
Relative Oka-Grauert principle on 1-convex spaces
- 16.45 – 17.30 Andreas Lind, Sundsvall:
A study of automorphisms of Danielewski surfaces
- 19.30 Dinner

Saturday

- 09.00 – 09.45 Jan Wiegerinck, Amsterdam:
Plurifinely plurisubharmonic and holomorphic functions
- 10.00 – 10.45 Ragnar Sigurðsson, Reykjavik:
Positive currents on thin pseudoconcave sets in \mathbb{C}^2
- 10.45 – 11.15 Coffee break
- 11.15 – 12.00 Armen Edigarian, Krakow:
Perron-Bremermann envelope via analytic disks
- 12.00 – 14.00 Lunch
- 14.00 – 14.45 Nikolay Shcherbina, Wuppertal:
On defining functions for unbounded pseudoconvex domains
- 14.45 – 15.15 Coffee break
- 15.30 – 18.00 Excursion
- 19.30 Dinner

Sunday

- 09.00 – 09.45 Ahmed Zeriahi, Toulouse:
A viscosity approach to the complex Monge-Ampère equation
- 10.00 – 10.45 Alexander Rashkovskii, Stavanger:
Approximation of plurisubharmonic singularities
- 10.45 – 11.15 Coffee break
- 11.15 – 12.00 Berit Kemppe, Umeå:
A plurisubharmonic ordering of measures

Participants

1	Ahmad Al Abdulaali	Stockholm University
	Mats Andersson	Chalmers, Göteborg
	Bo Berndtsson	Chalmers, Göteborg
	Stefan Borell	Mid Sweden University, Sundsvall
	Linus Carlsson	Umeå University
	Urban Cegrell	Umeå University
	Armen Edigarian	Jagiellonian University, Krakow
	Jens Forsgård	Stockholm university
	Anders Fällström	Umeå university
10	Lisa Hed	Umeå University
	Marius Irgens	NTNU, Trondheim
	Petter Johansson	Stockholms universitet
	Berit Kemppe	Umeå University
	Frank Kutzschebauch	University of Bern
	Aron Lagerberg	Chalmers, Göteborg
	Andreas Lind	Mid Sweden University, Sundsvall
	Sam Lodin	Mid Sweden University, Sundsvall
	Johannes Lundqvist	Stockholm University
	Richard Lärkäng	Chalmers, Göteborg
20	Erik Løw	University of Oslo
	Mikael Passare	Stockholm Universitty
	Håkan Persson	Umeå University
	Jasna Prezelj	University of Ljubljana
	Alexander Rashkovskii	University of Stavanger
	Hossein Raufi	Chalmers, Göteborg
	Nikolay Shcherbina	Bergische Universität, Wuppertal
	Ragnar Sigurðsson	University of Iceland, Reykjavik
	Jacob Sznajdman	Chalmers, Göteborg
	Jan Wiegerinck	University of Amsterdam
30	Frank Wikström	Lund University
	Elizabeth Wulcan	Chalmers, Göteborg
	Yang Xing	Lund University
	Ahmed Zeriahi	Université Paul Sabatier, Toulouse
	Per Åhag	Umeå University

Abstracts

Bo Berndtsson

A Brunn-Minkowski theorem for Fano manifolds and uniqueness for Kähler-Einstein equations

The Brunn-Minkowski theorem is an inequality for volumes of certain convex sets. It has an addendum saying that equality can only occur if the sets are translates of each other. I will give an analogous inequality for volumes of compact complex manifolds with positive Ricci curvature, so called Fano manifolds. Here equality can hold only if the metrics are related via the flow of a holomorphic vector field, i.e. are complex 'translates' of each other. I use this to prove a generalization of the Bando-Mabuchi uniqueness theorem.



Armen Edigarian

Perron-Bremermann envelope via analytic disks

Let B be the unit ball and let f be a function defined on the boundary bB . We construct the Perron-Bremermann envelope of f by proper holomorphic mappings from the unit disc D to B . We apply the result to polynomial hull of compact subsets of bB .



Berit Kemppe

A plurisubharmonic ordering of measures

In my PhD thesis I describe an ordering of measures induced by negative plurisubharmonic functions in the following sense.

$$\mu \preceq \nu \iff \int_{\Omega} \varphi d\mu \geq \int_{\Omega} \varphi d\nu, \quad \forall \varphi \in PSH^-(\Omega)$$

Here Ω is a bounded hyperconvex domain in \mathbb{C}^n and $PSH^-(\Omega)$ denotes the set of negative plurisubharmonic functions on Ω . The measures considered are finite positive regular Borel measures on Ω . In this talk I will discuss some interesting properties of this plurisubharmonic ordering.



Andreas Lind

A study of automorphisms of Danielewski surfaces

In the early 1990s Andersén and Lempert studied automorphisms of \mathbb{C}^n . They showed, and therefore answering a question by Rosay and Rudin, that the group generated by overshoots is dense, in the compact-open topology, in the automorphisms group of \mathbb{C}^n . This work was continued by Forstneric and Rosay, who used vector fields, and their flows, to study biholomorphic mappings, and in particular automorphisms of \mathbb{C}^n . Varolin extended Forstneric and Rosays work to complex manifolds. Varolin used vector fields to define the so called *density property* (and some variants of this property), and in



several papers Varolin proved that many interesting manifolds do have the density property. Varolin also showed some applications for manifolds with density property. The last three years Kutzschebauch and Kaliman have worked on finding examples of manifolds with density property, and they also have proved some applications.

In this talk I will extend Andersén and Lempert's work directly to Danielewski surfaces. A Danielewski surface is a smooth hypersurface D_p in $\mathbb{C}_{x,y,z}^3$ defined by $\{xy - p(z) = 0\}$, where $p \in \mathbb{C}[z]$ is a polynomial with simple zeros. I will give a natural definition of an overshear on D_p , and give an indication how to prove that the overshear group is dense in the component of the identity of the automorphisms group of D_p . This is a part of my Ph.D. thesis, and a paper of myself and Kutzschebauch.

Hopefully I also will have time to give a short survey of which manifolds have the density property, and give some open problems in this area of research.

Jasna Prezelj

Relative Oka-Grauert principle on 1-convex spaces

We present the relative Oka-Grauert principle for holomorphic submersions over 1-convex spaces using conic neighbourhoods of holomorphic sections over 1-convex spaces.

As an application we have the generalized Oka-Grauert principle for 1-convex manifolds: Every continuous mapping X to Y from a 1-convex manifold X to a complex manifold Y which is already holomorphic on a neighborhood of the exceptional set is homotopic to a holomorphic one provided that either Y satisfies the CAP property or we are free to change the complex structure on X .



Alexander Rashkovskii

Approximation of plurisubharmonic singularities

We consider classes of isolated singularities of plurisubharmonic functions that can be approximated by analytic singularities with control over the residual Monge-Ampère masses. They are characterized in terms of Green functions for Demailly's approximations and relative types. Furthermore, they are shown to appear when studying graded families of ideals of analytic functions and the corresponding asymptotic multiplier ideals. The types relative to such singularities are represented as lower envelopes of weighted divisorial valuations and analytic disk functionals.



Nikolay Shcherbina

On defining functions for unbounded pseudoconvex domains

We show that there exist an unbounded strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$ and a Wermer type set $\mathcal{E} \subset \Omega$ such that each defining function for Ω fails to be strictly plurisubharmonic on \mathcal{E} . We also prove a Liouville theorem for Wermer type sets.



Ragnar Sigurðsson

Positive currents on thin pseudoconcave sets in \mathbb{C}^2

This lecture is a report on a joint work with Zbigniew Slodkowski, University of Illinois at Chicago.



We are concerned with existence of a positive closed $(1, 1)$ -current on $\mathbb{D} \times \mathbb{C}$, where \mathbb{D} denotes the unit disc in the complex plane \mathbb{C} , supported by a prescribed relatively closed subset K of $\mathbb{D} \times \mathbb{C}$.

Such a current is of the form $T = \frac{i}{2} \partial \bar{\partial} \psi$, where ψ is a plurisubharmonic function on $\mathbb{D} \times \mathbb{C}$ and the support K of T must be a pseudoconcave subset of $\mathbb{D} \times \mathbb{C}$, i.e. the open set $\mathbb{D} \times \mathbb{C} \setminus K$ is pseudoconvex.

We assume that all the fibers $K_z = \{w : (z, w) \in K\}$ are totally disconnected and show, that with some additional topological conditions on K , a family $(m_z)_{z \in \mathbb{D}}$ of measures with support m_z contained in K_z can be constructed such that the function ψ is given as

$$\psi(z, w) = \int_{K_z} \ln |w - \zeta| dm_z(\zeta).$$

The construction is based on many results from the theory of analytic multifunctions and the theory of finely subharmonic and finely holomorphic functions.

Jan Wiegerinck

Plurifinely plurisubharmonic and holomorphic functions

As is well known, the fine topology is the weakest topology on domains in \mathbb{R}^n that makes all subharmonic functions continuous. It allows naturally for finely subharmonic — and in the 2-dimensional case also finely holomorphic — functions.



In \mathbb{C}^n the plurifine topology, which makes all plurisubharmonic functions continuous, is challenging. In this setting we introduce a weak and a strong concept of plurifinely plurisubharmonic and plurifinely holomorphic functions. Strong will imply weak, but it is unknown whether the two concepts are the same.

In this lecture we will discuss the plurifine topology and present our results on plurifinely plurisubharmonic and holomorphic functions. All this is joint work, partly with Said El Marzguioui, and partly with Mohamed El Kadiri and Bent Fuglede, and it includes

- Every bounded finely plurisubharmonic function can be locally (in the pluri-fine topology) written as the difference of two usual plurisubharmonic functions. As a consequence finely plurisubharmonic functions are continuous with respect to the pluri-fine topology.
- The $-\infty$ sets of finely plurisubharmonic functions are pluripolar, hence graphs of finely holomorphic functions are pluripolar.
- A function f is weakly plurifinely plurisubharmonic if and only if it is locally bounded from above in the plurifine topology and $f \circ h$ is finely subharmonic for all complex affine-linear maps h .
- Weak plurifine plurisubharmonicity and weak plurifine holomorphy are preserved under composition with weakly plurifinely holomorphic maps.

Ahmed Zeriahi

A viscosity approach to the complex Monge-Ampère equation



Degenerate complex Monge-Ampère equations on compact Kähler manifolds have been recently intensively studied using tools from pluripotential theory.

We develop an alternative approach based on the concept of viscosity solutions and compare systematically viscosity concepts with pluripotential theoretic ones.

This approach works only for a rather restricted type of degenerate complex Monge-Ampère equations. Nevertheless, we prove that the local potentials of the singular Kähler-Einstein metrics constructed previously by the authors are continuous plurisubharmonic functions. They were previously known to be locally bounded.

Another application is a lower order construction with a C^0 -estimate of the solution to the Calabi conjecture which does not use Yau's celebrated theorem.