NORDAN 2015, REYKJAVÍK
– Titles and abstracts –

MATS ANDERSSON

Chalmers University of Technology and University of Gothenburg, Sweden

ON THE $\bar\partial$-EQUATION ON A PURE-DIMENSIONAL ANALYTIC SPACE

ABSTRACT: Let X be a, possibly not reduced, analytic space of pure dimension n. I will discuss intrinsic notions of smooth forms, currents and the dbar-equation on X. I will indicate how one, by means of integral formulas, under suitable conditions on the right hand side can solve the $\bar\partial$-equation. In this way, one can also define fine sheaves A_k of $(0,k)$-currents, $k = 0,\dots,n$, that admit a resolution of the structure sheaf $\mathscr{O}^X$. The sheaves A_k coincide with the sheaves of smooth forms generically on X; more precisely, where the underlying reduced space is smooth and $\mathscr{O}^X$ is Cohen-Macaulay. This is a joint work, in progress, with Richard Larkang.

ZBIGNIEW BŁOCKI

Jagiellonian University, Kraków, Poland

BERGMAN KERNEL AND THE KOBAYASHI PSEUDODISTANCE IN CONVEX DOMAINS

ABSTRACT: We will discuss lower and upper bounds for the Bergman kernel (on the diagonal) for convex domains in $\mathbb{C}^n$ in terms of the reciprocal of the volume of the Kobayashi indicatrix. Several computational results for complex ellipsoids will also be given. This is a joint work with Wlodzimierz Zwonek.

RAFAŁ CZYŻ

Jagiellonian University, Kraków, Poland

DELTA-PLURISUBHARMONIC FUNCTIONS

ABSTRACT: The aim of this talk is to study the vector space of delta-plurisubharmonic functions (i.e. the vector space of differences of negative plurisubharmonic functions) with the vector ordering generating by the cone of plurisubharmonic functions. It will be proved that for $n > 1$ it is not a Riesz space. Next two different approaches to define the complex Monge-Ampere operator on this vector space will be presented and some Dirichlet problems will be solved. Finally several open problems related to this topic will be discussed.

Franc Forstnerič

University of Ljubljana, Slovenia

Complete conformal minimal surfaces with Jordan boundaries in $\mathbb{R}^n$

Abstract: (Joint work with A. Alarcón, B. Drinovec Drnovšek and F.J. López.) Let M be a compact bordered Riemann surface and $u_0 : M \to \mathbb{R}^n$ a conformal minimal immersion for some $n \geq 3$. Given a number $\varepsilon > 0$ we construct a continuous map $u : M \to \mathbb{R}^n$ such that (a) $\sup_M |u - u_0| < \varepsilon$, (b) u restricted to the interior $M \setminus bM$ is a complete conformal minimal immersion, and (c) the boundary $u(bM)$ is a finite union of Jordan curves in $\mathbb{R}^n$. If $n \geq 5$ then u can be chosen a topological embedding. This gives an essentially optimal (negative) answer to Calabi's Conjecture from 1965. Furthermore, we construct proper conformal minimal immersions $M \to D$ to any mean-convex domain $D \subset \mathbb{R}^3$. In particular, the complement of a properly embedded minimal surface in $\mathbb{R}^3$ admits plenty of proper minimal surfaces parametrized by any given bordered Riemann surface. The main toool is an approximate solution of the Riemann-Hilbert boundary value problem for holomorphic null curves in $\mathbb{C}^n$.

Frank Kutzschebauch

Universität Bern, Switzerland

Flexibility in Complex Analysis - the search for examples

Abstract: We will recall recently introduced notions of flexibility in Complex Analysis, as well as their algebraic analogues. Since the exact relations between these notions are not understood yet, one is in need of new examples. We propose to clarify the situation for smooth affine algebraic surfaces and concentrate on describing complete algebraic vector fields on them. The group generated by their time-t maps we propose to call $AAut_{hol}$- the group of algebraically holomorphic automorphisms. In the talk we present a classification/characterization of those surfaces on which $AAut_{hol}$ acts (quasi) transitively. Moreover our methods allow to classify all complete algebraic vector fields on each particular surface in the list. The presented results are from several joint works with Kaliman, Leuenberger and Liendo and work of Leuenberger alone.

Finnur Lárusson

University of Adelaide, Australia

The Oka principle in geometric invariant theory

Abstract: I will report on joint work with Frank Kutzschebauch and Gerald W. Schwarz on the Oka principle in geometric invariant theory. Our first paper was completed in 2013 and is to appear in Crelle. Two further papers with stronger results were posted in March 2015. They are all available on the arXiv, arxiv:1303.4779, arXiv:1503.00794 and arXiv:1503.00797.

The set-up of holomorphic geometric invariant theory is a reductive complex Lie group G acting holomorphically on a Stein manifold X. The orbit space X/G is typically ugly, but a more sophisticated quotient $X//G$, called the categorical quotient, exists as a normal Stein space. The gist of our results is that X is determined as a G-manifold by less information than we have any right to expect.

I will assume minimal acquaintance with geometric invariant theory. I will give plenty of background and emphasise general ideas over technical details.

Norm Levenberg

Indiana University, Bloomington, United States

Extremal functions in pluripotential theory

Abstract: We discuss notions of weighted and unweighted extremal (ω) plurisubharmonic functions associated to closed subsets of $\mathbb{C}^n$ ($\mathbb{P}^n$). Our goal is to give an explicit formula for one of these functions associated to a particular set and weight. The interest lies in the tools we utilize. This is joint work-in-progress with L. Bos, S. Ma'u and F. Piazzon.

Karl Oeljeklaus

Aix-Marseille University, France

Schottky group actions on homogeneous-rational manifolds and their non-Kähler compact quotients

Abstract: In the classical situation, a Schottky group action on the Riemann sphere is the action of a discrete subgroup $\Gamma \in SL(2,\mathbb{C})$ such that the quotient of the maximal proper domain is a Riemann surface of genus $g \geq 2$. As an abstract group, Γ is a non-abelian free group with g generators.

In the past Nori, Larusson and Verjowsky-Seade generalized the above setting to higher dimensions and studied Schottky group actions on odd-dimesional projective spaces $\mathbb{P}_{2n+1}(\mathbb{C}), n \in \mathbb{N}$. In this talk we investigate the problem which other homogeneous rational manifolds $X = S/P$ admit this kind of action and give several new examples. Furthermore we discuss properties of the non-Kähler compact complex manifolds which appear as Γ-quotients of the maximal proper domains.

Dan Popovici

Université Paul Sabatier, Toulouse, France

Positivity and Duality in Bidegrees $(1,1)$ and $(n-1,n-1)$ on Compact Complex Manifolds

Abstract: Let X be a compact complex manifold of dimension n. Starting from the duality between the Bott-Chern cohomology of bidegree $(1,1)$ and the Aeppli cohomology of bidegree $(n-1,n-1)$, we will explain how Lamari's positivity criterion between the pseudoeffective cone (introduced by Demailly in 1992) and the closure of the Gauduchon cone (that we introduced in 2013) can be used to tackle two long-standing conjectures. In particular, we will explain our proof of the qualitative part of Demailly's Transcendental Morse Inequalities Conjecture for a difference of two nef Bott-Chern cohomology classes on a compact Kähler manifold.

Jasmin Raissy

Université Paul Sabatier, Toulouse, France

Wandering Fatou components in dimension two

Abstract: The Fatou set of a holomorphic endomorphism of a complex manifold is the largest open set where the family iterates of the map form a normal family, and a Fatou component is a connected component of the Fatou set. In dimension one, Sullivan's Non Wandering Domain Theorem asserts that every Fatou component of a rational map is eventually periodic. Several classes of counter-examples have been found and studied for entire transcendental function in dimension one, but the question of the existence wandering Fatou components for polynomial endomorphisms in higher dimension remained open. We show, starting from an original idea of M. Lyubich and using techniques of parabolic bifurcation, that there exist polynomial endomorphisms of $\mathbb{C}^2$ with a wandering Fatou component. These maps are polynomial skew-products, and can be chosen to extend to holomorphic endomorphisms of the complex projective space. (Joint work with M. Astorg, X. Buff, R. Dujardin and H. Peters)

Hossein Raufi

Chalmers University of Technology and University of Gothenburg, Sweden

Log concavity for matrix-valued functions, and a matrix-valued Prékopa theorem

Abstract: Guided by positivity properties for metrics on holomorphic vector bundles, we define what it means for a matrix valued function to be log concave. After discussing some simple examples and properties, we introduce a matrix valued Prékopa theorem. We end by sketching the ideas behind the proof of this theorem, using a Paley-Wiener type of theorem and positivity properties of direct image bundles.

Tyson Ritter

University of Oslo, Norway

A soft Oka principle for proper embeddings of Riemann surfaces into $(\mathbb{C}^*)^2$

Abstract: Let X be an open Riemann surface. We prove an Oka property on the approximation and interpolation of continuous maps from X into $(\mathbb{C}^*)^2$ by proper holomorphic embeddings, provided we allow a deformation of the complex structure on X. This generalises and strengthens a recent result of Alarcón and López on proper embeddings of open Riemann surfaces into $\mathbb{C}^2$.