NORDAN 2016: Several Complex Variables

Location: room ??
Abstracts

A note on some fiber-integrals $\int_{f=s}\ \rho.(\omega/df)\wedge(\overline{\omega/df})$

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We remark that the study of a fiber-integral of the type

$$F(s) := \int_{f=s} \rho.(\omega/df)\wedge(\overline{\omega/df})$$

either in the local case where $\rho \equiv 1$ around 0 is $\mathcal{C}^\infty$ and compactly supported near the origin which is a singular point of $\{f = 0\}$ in $\mathbb{C}^{n+1}$, or in a global setting where $f : X \to D$ is a proper holomorphic function on a complex manifold X, smooth outside $\{f = 0\}$ with $\rho \equiv 1$ near $\{f = 0\}$, for given holomorphic $(n+1)-$forms ω and ω', that a better control on the asymptotic expansion of F when $s \mapsto 0$, is obtained by using the Bernstein polynomial of the "frescos" associated to f and ω and to f and ω' (a fresco is a "small" Brieskorn module corresponding to the differential equation deduced from the Gauss-Manin system of f at 0) than to use the Bernstein polynomial of the full Gauss-Manin system of f at the origin. We illustrate this in the local case in some rather simple (non quasi-homogeneous) polynomials, where the Bernstein polynomial of such a fresco is explicitly evaluated.

Degrees of polynomial appoximation in holomorphic Carleman classes

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In this talk, we extend results of M.S. Baouendi and C. Goulaouic (Ann. Inst. Fourier, 1971 ; Trans. Amer. Math. Soc, 1974), obtained for compacts of $\mathbb{R}^N$ with analytic boundary. If K is a compact of $\mathbb{C}^N \backsimeq \mathbb{R}^{2N}$, $(N \geq 1)$, $\mathcal{H}_M(K)$ is the space of $\overline{\partial}$-Whitney jets on K which are of class $\{M\}$, where $M(t) = t^t e^{t\mu(t)}$, $t >> 0$ and μ belongs to a Hardy field. We prove that a jet

$F := (F^\alpha)_{\alpha\in\mathbb{N}^{2N}} \in \mathcal{H}_M(K)$ if and only if there exists a constant $C > 0$, such that

$$\lim_{n\to\infty} d_n(F^\alpha, K)\exp(C\overline{\omega}_{K,M}(n)) = 0, \quad \text{for all} \quad \alpha \in \mathbb{N}^{2N},$$

where $d_n(\cdot, K)$ is the distance, for the uniform norm on K to the complex vectorial space of polynomials of degree at most n, and where $\overline{\omega}_{K,M}$ is a weight depending on the class $\{M\}$ and K. If K is Whitney-regular

$$\mathcal{H}_M(K) \backsimeq \big\{f \in \mathcal{E}^\infty(K) \cap \mathcal{O}(\dot{K}) : \\ \exists C > 0,\ \exists \rho > 0,\ \|D^\alpha f\|_K \leqslant C\rho^{|\alpha|}M(|\alpha|),\ (\forall \alpha \in \mathbb{N}^N)\big\}.$$

In this situation, $f \in \mathcal{H}_M(K)$ if and only if $\lim\limits_{n\to\infty} d_n(f, K)e^{C\overline{\omega}(n)} = 0$, where $C > 0$ and $\overline{\omega}$ is a weight depending on $\{\mathrm{M}\}$. Finally, we annonce similar results in the situation where K is a compact of some Stein manifold. A crucial role is played by a new geometric criteria: the Łojasiewicz-Siciak condition for the Green function of K. This is a joint work with Boutayeb El Ammari and Laurent P. Gendre.

On the nonvanishing of abstract Cauchy-Riemann cohomology groups

Judith Brinkschulte
University of Leipzig, Germany

In this talk, I will discuss a joint work with C.D. Hill and M. Nacinovich. Namely we considered abstract smooth CR structures on a smooth manifold M, and the associated global abstract cohomology groups $H^{p,q}(M)$, which are the analogues of the Dolbeault cohomology groups. We have shown that some of these global cohomology groups must be infinite dimensional, or non Hausdorff, whenever one has a certain condition on the Levi form of the CR structure. What makes our results curious is that the required condition on the Levi form needs to be satisfied only at a single (mircro-local) point on M; yet the conclusion is global.

High-power asymptotics of weighted harmonic Bergman kernels

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The asymptotics of the weighted Bergman kernels with respect to the weight $|r|^\alpha$, where r is a defining function for a smoothly bounded strictly pseudoconvex domain and $\alpha \to +\infty$, play prominent role in mathematical physics (Berezin quantization) as well as in complex geometry (Donaldson's balanced metrics); the standard tool for their derivation is the famous description of the boundary singularity of the Bergman kernel due to Fefferman, combined with a construction due to Forelli and Rudin. The talk will describe why it is noteworthy to study the analogous asymptotics also for the Bergman kernels for harmonic functions, and will give a complete answer for the case of radial weights on the ball and horizontal weights on the upper half-space. The proofs actually proceed by relating the problem to the holomorphic case mentioned above, but on a different domain.

Invariant measures for birational surface maps defined over number fields

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Iterating a birational selfmap of the complex projective plane can lead to very interesting dynamics. When the dynamical degree is larger than one, it is expected that there exists a unique measure of maximal entropy, and that this measure is obtained as the intersection of two positive closed currents. Unfortunately, this intersection is only known to be well-defined under hypotheses that are often hard to verify. I will report on joint work with Paul Reschke, where we prove that if the map has rational (or algebraic) coefficients, then the intersection is well defined and the complex dynamics is under control.

An Oka principle for simultaneous standardization of n-tuples of points

Frank Kutzschebauch
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It is an easy exercise to show that the holomorphic automorphism group $Aut_{hol}(\mathbb{C}^n)$ $n \geq 2$ acts transitively on ordered N-tuples $(z_1, z_2, \dots z_N)$ of pairwise disjoint points in $\mathbb{C}^n$. If the points depend holomorphically on a Stein parameter $w \in W$ we ask whether the automorphism of $\mathbb{C}^n$ moving the N-tuple $(z_1(w), z_2(w), \dots z_N(w))$ to a fixed N-tuple $(z_1, z_2, \dots z_N)$ can be chosen holomorphically depending on the parameter w. We prove an Oka principle saying that the obstruction for this is of purely topological nature. Our theorem (which is true not only for $\mathbb{C}^n$ but for any Stein manifold with the density property) can be interpreted as a result similar to Grauerts Oka principle, but instead for the principal bundle of a complex Lie group in Grauerts case for a principal bundle of certain infinite-dimensional Frechet groups in our case. This is a joint work with Ramos Peon.

Intersection theory and relative fundamental classes

Jón Ingólfur Magnússon
University of Iceland

We will describe an intersection theory for analytic cycles in a complex manifold. Main properties will be explained, in particular with respect to analytic families of cycles and their relative fundamental classes. This is a joint work with Daniel Barlet.

Automorphism groups of Levi degenerate hypersurfaces in $\mathbb{C}^3$

Francine Meylan
University of Fribourg, Switzerland

We give a classification of smooth real hypersurfaces of finite Catlin multitype in $\mathbb{C}^3$ which admit nonlinear infinitesimal automorphisms. The results are complete on the level of weighted homogenous polynomial models. As a consequence, we prove a sharp 1-jet determination result in the general case.

We also identify a common source of such vector fields. They all arise by pulling back a symmetry of a suitable hyperquadric in $\mathbb{C}^K$, $K \geq 3$, by a holomorphic mapping from $\mathbb{C}^3$ to $\mathbb{C}^K$. This is a joint work with Martin Kolar.

Duality between the pseudoeffective and the movable cone on a projective manifold.

David Witt Nyström
Chalmers University of Technology and the University of Gothenburg

The structure of projective algebraic manifolds is to a large extent governed by the geometry of its cones of divisors or curves. In the case of divisors, two cones are of primary importance: the cone of ample divisors and the cone of effective divisors (and the closure of these cones as well). These cones have natural transcendental analogues, namely the cone of Kähler classes (called the Kähler cone) and the cone of pseudoeffective $(1,1)$-classes (called the pseudoeffective cone).

I will discuss my recent proof of a conjecture of Boucksom-Demailly-Paun-Peternell which says that on a projective manifold the pseudoeffective cone is dual to the cone of movable classes. A celebrated result of Demailly-Paun showed that the Kähler cone is determined by analytic cycles and the Hodge structure of the manifold. A consequence of the duality theorem is that on a projective manifold the pseudoeffective cone is similarly determined by the analytic cycles and the Hodge structure of the manifold together with all its modifications.

Positivity and Duality on Compact Complex Manifolds

Dan Popovici
Paul Sabatier University, Toulouse, France

Given a compact complex manifold X of dimension n, we shall explain how the duality between Demailly's pseudoeffective cone of Bott-Chern cohomology classes of closed positive $(1,1)$-currents and our Gauduchon cone of Aeppli cohomology classes of $(n-1)^{st}$ powers of Gauduchon metrics can be used, together with a natural way of estimating from below the integral of a quantity involving the solution of a Monge-Ampère equation provided by Yau's

theorem, to completely prove the qualitative part and give a partial affirmative answer to the quantitative part of Demailly's Transcendental Morse Inequalities Conjecture for differences of two nef classes. If time permits, we shall explain how a complete solution to this conjecture would contribute to tackling the conjecture we have proposed in the non-Kähler context predicting that every $\partial\bar{\partial}$-manifold should carry a balanced metric.

Analytic and plurisubharmonic singularities

Alexander Rashkovskiĭ
University of Stavanger, Norway

The subject of the talk is how good one can approach isolated plurisubharmonic singularities by analytic ones. The following two (related) open problems will be discussed: existence of a plurisubharmonic function with the zero Lelong number and positive residual Monge-Ampère mass, and control over the residual Monge-Ampère masses of analytic approximations.

On vanishing results for L^2-Dolbeault cohomology on pseudoconvex complex spaces

Martin Sera
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Using Grauert's bumping method, it was proven that L^2-Dolbeault cohomology groups of a strictly pseudoconvex bounded domain in a complex manifold are isomorphic to Dolbeault cohomology groups with respect to locally square integrable forms. We generalize this to strictly pseudoconvex domains in complex spaces. For this, we prove vanishing theorems for L^2-Dolbeault cohomology groups on pseudoconvex domains of analytic sets in $\mathbb{C}^n$, which generalize results by T. Ohsawa, W. Pardon and M. Stern.

A characterization of polynomials in complex and non-archimedean dynamics

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In 1960s Hans Brolin initiated systematic application of potential-theoretic methods in the dynamics of holomorphic polynomials. Among other things, he proved the now-famous equidistribution theorem: for a polynomial f of degree greater than 1 the preimages, under successive iterates of f, of a Dirac measure at an arbitrary point of the complex plane (except at most two so-called exceptional points) converge weakly to the equilibrium measure of the Julia set for f. In 1980s a similar result (about convergence of preimages of quite general probabilistic measures) was proved for a rational map f of degree greater than 1. The limit measure obtained in this case (called the balanced measure) is also supported on the Julia set for f, but does not have to be its equilibrium measure. In fact, A.O. Lopes proved (using dynamical properties of Julia sets) that equality of these two measures (under suitable assumptions on f, also making precise the notion of the equilibrium measure for the Julia set) implies that f is a polynomial. In 2010 we obtained a proof of Lopes's theorem (under slightly weaker assumptions) using only classical and weighted potential theory. In this talk I will present a recent extension of this result, namely a characterization of polynomials among rational functions, up to rational functions having potentially good reductions as exceptions, on the projective line over an algebraically closed field of any characteristic that is complete with respect to a non-trivial and possibly non-archimedean absolute value. I will introduce basic notions of non-archimedean dynamics and discuss possible cases. This is joint work with Yusuke Okuyama from Kyoto Institute of Technology.

Boundary behavior of invariant functions on planar domains

Maria Trybuła
Adam Mickiewicz University, Poznań, Poland

In the talk I will show the precise behavior of Carathéodory, Kobayashi and Bergman metrics and the corresponding distances near smooth boundary points of planar domains under different assumptions of regularity. Some applications will be given. The lecture is based on a joint paper with N. Nikolov and L. Andreev