NORDAN 2018 - Stavanger, Norway, 25.–27. May 2018

Conference Program

Pre-conference seminar, University of Stavanger, Ullandhaug

Friday 25. May

- 9:30-10:15 Miroslav Engliš: Reproducing kernels and distinguished metrics
- 10:30-11:15 Caterina Stoppato: Slice regular functions of a hypercomplex variable
- 11:30-12:15 Yaacov Kopeliovich: Realizing irreducible representations of symmetric groups through theta functions
- 12:30-13:30 Lunch and refreshments
- 13:45-16:00 Bus to conference venue

Main conference, Spa-Hotell Velvære, Hjelmeland

Friday 25. May

- 16:30-17:15 Filippo Bracci: Strange Fatou components of automorphisms of \mathbb{C}^2
- 17:20-18:05 Trung Tuyen Truong: Can we intersect a line in the plane with itself?
- 18:10-18:55 Séverine Biard: Estimates for the complex Green operator on CRsubmanifolds of hypersurface type: symmetry and interpolation 19:30 Dinner

Saturday 26. May

- 08:45-09:30 August Tsikh: Some collateral problems of complex analysis and tropical geometry
- 09:45-10:30 Samuele Mongodi: On the classification of weakly complete surfaces
- 10:30-11:00 Coffee break
- 11:00-11:45 Stéphanie Nivoche: A new proof of a problem of Kolmogorov on ϵ -entropy
- 12:00-12:45 Maria Trybuła: On the Gromov non-hyperbolicity of certain domains in \mathbb{C}^n
- 13:00-14:00 Lunch
- 15:00-17:00 Excursion: boat trip along Jøsenfjorden 19:30 Main conference dinner

Sunday 27. May

- 08:45-09:30 David Witt Nyström: Deforming a Kähler manifold to the normal cone of a subvariety
- 09:45-10:30 Tat Dat Tô: Fully non-linear parabolic equations on compact Hermitian manifolds
- 10:30-11:00 Coffee break
- 11:00-11:45 Jan Wiegerinck: Domains of existence for finely holomorphic functions
- 12:15-13:00 Lunch
- 13:00-13:30 Check out from hotel

14:15 Departure by ferry from Hjelmeland, direct to Fiskepiren, Stavanger

Titles and Abstracts - Pre-conference seminar

Speaker: Miroslav Engliš

Title: Reproducing kernels and distinguished metrics

Abstract: Two classical distinguished Hermitian metrics on a complex domain are the Bergman metric, coming from the reproducing kernel of the space of square-integrable holomorphic functions, and the Poincaré metric, i.e. a Kaehler-Einstein metric with prescribed (natural) behaviour at the boundary. In the setting of compact Kaehler manifolds rather than domains, the so-called balanced metrics were introduced some time ago by S. Donaldson, building on earlier works of S.-T. Yau and G. Tian.

The talk will discuss the questions of existence and uniqueness of balanced metrics on (noncompact) complex domains, where some answers are yet unknown nowadays even for the simplest case of the unit disc.

Speaker: Yaacov Kopeliovich

Title: Realizing irreducible representations of symmetric groups through theta functions

Abstract: In this talk I will explain how theta functions evaluated at certain points realize symmetric group representations.

Speaker: Caterina Stoppato

Title: Slice regular functions of a hypercomplex variable

Abstract: Since the 1930s, several function theories have been introduced over the algebra of quaternions and other hypercomplex algebras. The aim of such constructions is to recover in higher dimensions the refined tools that are available in the complex case through the theory of holomorphic functions. The peculiarities of the noncommutative setting are reflected in the different theories introduced.

Among these approaches to hypercomplex analysis, the one Gentili and Struppa set out in 2006 for the quaternions has rapidly developed into a full-fledged theory. It is the object of current research along with its generalization to alternative *-algebras, introduced by Ghiloni and Perotti in 2011.

The talk will overview the main features of the theory and its applications to open problems from other areas of mathematics. Time permitting, some recent results will be presented in more detail.

Titles and Abstracts - Main conference

Speaker: Séverine Biard

Title: Estimates for the complex Green operator on CR-submanifolds of hypersurface type: symmetry and interpolation

Abstract: Although the complex Green operator on CR-submanifolds of hypersurface type is naturally compared to the $\bar{\partial}$ -Neumann operator on pseudoconvex domains, some of its properties differ. Mainly, compactness estimates hold for forms of symmetric bidegrees but those estimates do not percolate up the tangential $\bar{\partial}$ -complex. However, in a joint work with E. Straube, we prove a result of interpolation of compactness estimates for the complex Green operator on smooth compact pseudoconvex orientable CR-submanifolds of \mathbb{C}^n of hypersurface type, giving an alternative to the percolation.

Speaker: Filippo Bracci

Title: Strange Fatou components of automorphisms of \mathbb{C}^2

Abstract: The classification of Fatou components for automorphisms of the complex space of dimension greater than 1 is an interesting and difficult task. Recent deep results prove that the one-dimensional setting is deeply different from the higher dimensional one. Given an automorphism F of \mathbb{C}^k , the first bricks in the theory that one would like to understand are invariant Fatou components, namely, those connected open sets U, completely invariant under F, where the dynamics of F is not chaotic. Among those, we consider attracting Fatou components, that is, those components on which the iterates of F converge to a single point. Attracting Fatou components can be recurrent, if the limit point is inside the component, or non-recurrent. Recurrent attracting Fatou components are always biholomorphic to \mathbb{C}^k , since it was proved by H. Peters, L. Vivas and E. F. Wold that in such a case the point is an attracting (hyperbolic) fixed point, and the Fatou component coincides with the global basin of attraction. Also, as a consequence of works of Ueda and Peters-Lyubich, it is known that all attracting non-recurrent Fatou components of polynomial automorphisms of \mathbb{C}^2 are biholomorphic to \mathbb{C}^2 . One can quite easily find non-simply connected non-recurrent attracting Fatou components in \mathbb{C}^3 (mixing a two-dimensional dynamics with a dynamics with non-isolated fixed points in one-variable).

In this talk I will explain how to construct a non-recurrent attracting Fatou component in \mathbb{C}^2 which is biholomorphic to $\mathbb{C} \times \mathbb{C}^*$. This "fantastic beast" is obtained by globalizing, using a result of F. Forstnerič, a local construction due to the speaker and Zaitsev, which allows to create a global basin of attraction for an automorphism, and a Fatou coordinate on it. The Fatou coordinate turns out to be a fiber bundle map on \mathbb{C} , whose fiber is \mathbb{C}^* , then the global basin is biholomorphic to $\mathbb{C} \times \mathbb{C}^*$. The most subtle point is to show that such a basin is indeed a Fatou component. This is done by

exploiting Pöschel's results about existence of local Siegel discs and suitable estimates for the Kobayashi distance.

Since attracting Fatou components are Runge, it turns out that this construction gives also an example of a Runge embedding of $\mathbb{C} \times \mathbb{C}^*$ into \mathbb{C}^2 . Moreover, this example shows an automorphism of \mathbb{C}^2 leaving invariant two analytic discs intersecting transversally at the origin.

The talk is based on a joint work with J. Raissy and B. Stensones.

Speaker: Samuele Mongodi

Title: On the classification of weakly complete surfaces

Abstract: In a series of papers with Z. Słodkowski (UIC) and G. Tomassini (SNS Pisa), we classified the weakly complete surfaces that admit a real analytic plurisubharmonic exhaustion function. These can fall into three classes:

- Modifications of Stein spaces,
- Surfaces proper over a (non-compact) Riemann surface, and
- Grauert-type surfaces.

The latter is a class of surfaces where the level sets of the exhaustion function are Levi flat and foliated with dense complex leaves. We have also investigated further the geometry of the surfaces of Grauert type and their weakly complete subdomains, obtaining an application to a form of the Levi problem for Hopf surfaces (which are a particular case of surfaces of Grauert type).

In this talk I would like to describe the classification result through the analysis of some examples and to focus on the properties of Grauert type surfaces; time permitting, I would also like to present some questions on the subject which are still unsolved.

Speaker: Stéphanie Nivoche

Title: A new proof of a problem of Kolmogorov on ϵ -entropy

Abstract: In the 80s, a Kolmogorov problem about the ϵ -entropy of a class of analytic functions was stated:

$$\lim_{\epsilon \to 0} \frac{H_{\epsilon}(A_K^D)}{\ln^{n+1}(1/\epsilon)} = \frac{2C(K,D)}{(2\pi)^n(n+1)!}$$

In 2004, this problem was solved by using techniques of pluripotential theory and in particular by proving a conjecture of Zakharyuta.

Here we will present a new proof of Kolmogorov's problem, independently of this conjecture. We will use the asymptotic behaviour of the Bergman kernel of a concentration operator and some properties of special analytic polyhedra.

Speaker: David Witt Nyström

Title: Deforming a Kähler manifold to the normal cone of a subvariety

Abstract: A classical construction in algebraic/complex geometry, known as degeneration to the normal cone, allows you to degenerate a variety to the normal cone of a subvariety. In this talk I will discuss what happens when you add a Kähler structure to this picture.

Speaker: Tat Dat Tô

Title: Fully non-linear parabolic equations on compact Hermitian manifolds

Abstract: We introduce a notion of parabolic *C*-subsolutions for parabolic equations, extending the theory of *C*-subsolutions recently developed by B. Guan and G. Szekélyhidi for elliptic equations, and also give some applications (joint work with Duong Hong Phong).

Speaker: Tuyen Trung Truong

Title: Can we intersect a line in the plane with itself?

Abstract: Bezout's theorem says that two distinct irreducible curves C_1 and C_2 in the projective plane intersect at $\deg(C_1) \deg(C_2)$ points when multiplicities are counted. This can be reinterpreted in complex analysis as follows: the wedge intersection $[C_1] \wedge [C_2]$ of the currents of integration $[C_1]$ and $[C_2]$ is a positive measure with mass $\deg(C_1) \deg(C_2)$. What happens if $C_1 = C_2 = C$? No answer had yet been given in the literature about what $[C] \wedge [C]$ should be. In this talk, I will show that if we allow a generalisation of measures, the so-called strong submeasures, then we can define $[C] \wedge [C]$ for all curves C in such a way to preserve Bezout's theorem. The result applies more generally to intersections of positive closed currents on compact Kähler manifolds and to dynamics.

Speaker: Maria Trybuła

Title: On the Gromov non-hyperbolicity of certains domains in \mathbb{C}^n

Abstract: Reporting on a joint work with Nikolai Nikolov (BAS) I will discuss the Gromov hyperbolicity of \mathbb{C} -convex domains in \mathbb{C}^n with respect to the Kobayashi distance. Specifically, I will prove that if Ω is a bounded \mathbb{C} -convex domain in \mathbb{C}^n , $n \geq 2$, and S is an affine complex hyperplane such that $\Omega \cap S$ is not empty, then $\Omega \setminus S$ is not Gromov hyperbolic. Next, I will localize this result for convex domains. Finally, I will show one result concerning Hartogs type domains, that is: if Ω is a bounded \mathbb{C} -convex domain in \mathbb{C}^n and φ is any function such that the set

$$\Omega_{\varphi} = \{ (z, w) \in \mathbb{C}^n \times \mathbb{C} : 0 \le |w| \le e^{-\varphi} \}$$

is open, then Ω_{φ} is not Gromov hyperbolic.

Speaker: August Tsikh

Title: Some collateral problems of complex analysis and tropical geometry

Abstract: I will talk about amoebas of complex algebraic sets. In terms of tropical geometry, we consider the question of constructing an order function on homology classes of complements to amoebas of higher codimension.

Speaker: Jan Wiegerinck

Title: Domains of existence for finely holomorphic functions

Abstract: Finely holomorphic functions are the natural generalisation of holomorphic functions in the setting of the fine topology. We recall definitions and properties of the fine topology and finely holomorphic functions and will study what remains of the well-known theorem of Weierstrass that every domain U in \mathbb{C} is a domain of existence. Roughly speaking, this says that every domain admits a holomorphic function that can nowhere be extended beyond U.

We will discuss joint work with Alan Groot and Bent Fuglede, showing that fine domains in \mathbb{C} with the property that they are Euclidean F_{σ} and G_{δ} , are in fact fine domains of existence for finely holomorphic functions. Moreover *regular* fine domains are also fine domains of existence. However, fine domains such as $\mathbb{C} \setminus \mathbb{Q}$ or $\mathbb{C} \setminus (\mathbb{Q} \times i\mathbb{Q})$, more specifically fine domains V with the property that their complement contains a non-empty polar set E that is of the first Baire category in its Euclidean closure K and that $(K \setminus E) \subset V$, are *not* fine domains of existence.

PARTICIPANTS

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Abdel Rahman Al-Abdallah (University of Regina) Alexander Rashkovskii (University of Stavanger) Andreas Andersson (Chalmers University of Technology) August Tsikh (Siberian Federal University) Azza Alghamdi (Uppsala University) Benedikt Magnússon (University of Iceland) Caterina Stoppato (Università di Firenze) David Witt Nyström (University of Gothenburg) Elizabeth Wulcan (Chalmers University of Technology) Erik Løw (University of Oslo) Filippo Bracci (Università di Roma "Tor Vergata") Irina Markina (University of Bergen) István Prause (University of Helsinki) Jan Wiegerinck (Universiteit van Amsterdam) Jimmy Johansson (University of Gothenburg) John Erik Fornæss (NTNU) Karim Rakhimov (The University of Pisa) Linus Bergqvist (Stockholm University) Long Li (Fourier Institute) Maria Trybuła (Adam Mickiewicz University in Poznań) Mats Andersson (Chalmers and University of Gothenburg) Michael Brünnig (University of Wuppertal) Miroslav Engliš (Mathematics Institute, Prague & Opava) Mitja Nedic (Stockholm University) Pascal J. Thomas (Institut de Mathématiques de Toulouse) Pavel Gumenyuk (University of Stavanger) Per Manne (Norwegian School of Economics) Richard Lärkäng (University of Gothenburg) Samuele Mongodi (Politecnico di Milano) Séverine Biard (University of Iceland) Stéphanie Nivoche (Côte d'Azur University, France) Tat Dat Tô (Université Toulouse III-Paul Sabatier, France) Tuyen Truong (University of Oslo) Tyson Ritter (University of Stavanger) Xu Wang (Norwegian University of Science and Technology) Yaacov Kopeliovich (University of Connecticut)