

### Program Nordan 2000 Ornsköldsvik



Supported by Mid Sweden University, Umeå University and NFR

Järnvägsgatan 6, Lecture hall 138 Örnsköldsvik Mid Sweden university

### Friday May 5.

16.00-		Coffee
17.00-17.45	Ozan Öktem, Stockholm	Extension of separately analytic functions and
18.00-18.45	Norman Levenberg, Auckland	applications to mathematical tomography.  Transfinite Diameter in C?N
19.30		Dinner
Saturday May 6.	ay 6.	
9.00-9.45	Lars Hörmander, Lund	Approximation of solutions of boundary problems and of entire functions
10.00-10.45	Erik Løw, Oslo	Solving the d-bar-equations in thin tubes and applications to mappings.
		Coffee
11.15-12.00 12.15-13.00	Bo Berndtsson, Göteborg Jan Boman, Stockholm	Complex analysis on currents A Paley-Wiener Theorem for the analytic wave front set.
		Lunch
15.00-15.45 16.00-16.45 17.15-18.00	Christer Kiselman, Uppsala Jockum Aniansson, Stockholm Nikolay Shcherbina, Göteborg	Lineal convexity, C-convexity and convexity Fisher kernels and Cauchy problems for the wave equation Hulls and Levi-flat surfaces.
19.00		Dinner
Sunday May 7.	7.	
9.00-9.45	Lars Filipsson, Gävle	PDE-preserving polynomial interpolation
		Coffee
10.00-10.45	Juhani Riihentaus, Joensuu	Subharmonic Functions: Non-Tangential and Tangential Roundary Rehavior
11.00-11.45	Hasse Carlsson, Göteborg	Harmonic analysis and several complex variables

quation.

#### Dinners etc.

Dinner on Friday will be at 19.30 at "Restaurant Mamma mia", located in the centre of Örnsköldsvik, Storgatan 6.

Lunch on Saturday will be in the locations of Mid Sweden university and Umeå univeristy. A house called "Arken" near the place where the seminars take place.

taking us there. The bus returns at 21.30. Dinner on Saturday will be at 19.00 at "Restaruant Varvsberget". This restaurant is located on the top of one of the mountains in Örnsköldsvik. There will be a bus from the hotel at 18.45

Lunch on Sunday will be outside the lecture hall.

#### Departure

each flight. There is a bus to the airport departing from the bus station nearby the hotel one hour before

### NORDAN 2000 IN ÖRNSKÖLDSVIK ABSTRACTS

### Friday May 5.

17.00-17.45 Ozan Öktem

Extension of separately analytic functions and applications to mathematical tomography.

Given a test function h in  $\mathbb{R}^2$ , we define the generalized exponential Radon transform  $R_p(h)$  of h as

$$R_{\rho}(h)(\omega,p) := \int_{x\cdot\omega=p} h(x)e^{\rho(\omega)x\cdot\omega^{\perp}} dm(x)$$

In the definition above,  $\rho \colon S^1 \to \mathbb{R}$  is a fixed function (which in our case is a rational function),  $\omega := (\cos \alpha, \sin \alpha)$  for  $0 \le \alpha < 2\pi$ ,  $\omega^1 := (-\sin \alpha, \cos \alpha)$ , and dm is the 1-dimensional Lebesgue measure on the line  $x \cdot \omega = p$ .

Problems of interest within mathematical tomography are to invert, study uniqueness properties, and to characterize the image of the operator  $R_{\rho}$  when it acts on various spaces.

In this joint work with Prof. Jan Boman, I will confine myself to the last problem, more precisely to characterize the image of  $R_p$  when it acts on the set of test functions in  $\mathbb{R}^2$ . I will show how extension properties of separately analytic functions play a central role when solving the range characterization problem of the generalized exponential Radon transform.

# 18.00-18.45 Norman Levenberg

Transfinite diameter in C'.

Let E and F be compact subsets of  $\mathbb{C}^N$  with E contained in F. We show that d(E) = d(F) if and only if F - E is pluripolar where d(E) denotes the transfinite diameter of E. The result is, admittedly, uninspiring; however, we think (hope!) the method of proof – which uses weighted pluripotential theory – will be of interest. This is joint work with Tom Bloom.

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Saturday May 6.

9.00-9.45 Lars Hörmander

Approximation of solutions of boundary problems and of entire functions Let  $\Omega \subset \mathbb{R}^n$  be open and convex,  $\omega = \{x \in \Omega, x_n = 0\} \neq \emptyset$ , and let P(D) and  $Q_1(D), \ldots, Q_J(D)$  be differential operators with constant coefficients such that the plane  $x_n = 0$  is not characteristic with respect to P(D). Set

$$C^{\infty}_{P,Q}(\Omega) = \{u \in C^{\infty}(\Omega); P(D)u = 0 \text{ in } \Omega, Q_1(D)u = \cdots = Q_J(D)u = 0 \text{ in } \omega\}.$$

The lecture will be devoted to the problem of deciding when  $C_{P,\vec{Q}}^{\infty}(\mathbb{R}^n)$  restricted to  $\Omega$  is dense in  $C_{P,\vec{Q}}^{\infty}(\Omega)$ . Duality and the Fourier-Laplace transformation reduces this to questions on approximating entire functions of exponential type by such functions of slower growth. Only partial answers to these questions can be given.

### 10.00-10.45 Erik Løw

Solving the d and  $\bar{\partial}$ -equations in thin tubess and applications to mappings

We construct a family of integral kernels for solving the  $\overline{\partial}$ -equation with  $C^k$  and Hölder estimates in thin tubes around totally real submanifolds in  $\mathbb{C}^n$  (theorems 1.1 and 3.1). Combining this with the proof of a theorem of Serre we solve the d-equation with estimates for holomorphic forms in such tubes (theorem 5.1). We apply these techniques and a method of Moser to approximate  $C^k$ -diffeomorphisms between totally real submanifolds in  $\mathbb{C}^n$  in the  $C^k$ -topology by biholomorphic mappings in tubes, by unimodular and symplectic biholomorphic mappings, and by automorphisms of  $\mathbb{C}^n$ .

(Joint with Franc Forstneric and Nils Øvrelid)

## 11.15-12.00 Bo Berndtsson

Complex analysis on currents (joint work with N Sibony)

We first discuss dbar-operators on positive (p,p)-currents, and the resulting definitions of holomorphic functions and forms on a current. We then prove solvability and  $L^2$ -estimates for the inhomogeneous dbar-equation for currents that are closed or more generally satisfies  $dd^cT \leq 0$ . The ideas are illustrated by examples relating to singular varieties, Bernstein-type problems on separate analyticity, concave domains and foliations.

### 12.15-13.00 Jan Boman

A Paley-Wiener Theorem for the analytic wave front set

It follows from the Paley-Wiener Theorem and the Phragmén-Lindelöf Theorem that the supporting function H for the support of a distribution u with compact support is given by

$$H(\eta) = \sup_{\xi \in \mathbb{R}^n} i_{\tilde{u}}(\xi + i\eta) = i_{\tilde{u}}(i\eta), \quad \eta \in \mathbb{R}^n; (1)$$

here  $i_{\ell}$  is the indicator function of  $\hat{u}_{\ell}$  which is defined as the upper semicontinuous regularization of the function  $\zeta \mapsto \lim_{\ell \to \infty} t^{-1} \log |\hat{u}(t\zeta)|$ . I will describe how information about the analytic singularities of u can be recovered from  $i_{\ell}(\zeta)$  for  $\zeta$  near  $\mathbb{R}^n$  as follows. For given  $\xi \in \mathbb{R}^n \setminus \{0\}$ , the supporting function  $H_{\xi}$  of the set

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 $K_{\xi} = \{x; (x,\xi) \in WF_{\mathcal{A}}(u)\}\$  of analytic singularities with cotangent direction  $\xi$  is given by

$$H_{\xi}(\eta) = \lim_{\delta \to +0} \varliminf_{s \to +0} \sup_{|\vec{\xi}-\xi| < \delta} i_{\vec{u}}(\vec{\xi} + is\eta)/s = \varlimsup_{s \to +0, \vec{\xi} \to \xi} i_{\vec{u}}(\vec{\xi} + is\eta)/s, \quad \eta \in \mathbb{R}^n.(2)$$

This statement is also valid for hyperfunctions with compact support. The convex hull of singsupp<sub>A</sub> u, the set of all analytic singularities of u, is of course equal to the convex hull of supp u, so it can already be determined by (1). It turns out that (2) gives no more information on singsupp<sub>A</sub> u than (1), for we show that the union of all  $K_{\xi}$  is equal to the convex hull of supp u. The proof consists of a topological argument involving existence of certain vector fields on spheres.

This is joint work with Lars Hörmander (Asian J. Mathvol3, no4, 1999).

## 15.00-15.45 Christer Kischman

Lineal convexity, C-convexity, and convexity.

In addition to ordinary convexity and pseudoconvexity, there are several interesting notions of convexity in complex geometry. A set is said to be lineally convex if its complement is a union of complex hyperplanes. An open set is said to be C-convex if its intersection with every complex line is either empty or contractible to a point. In my lecture I will discuss a result which connects several notions of convexity: I will show that lineally convex Hartogs domains define a convex set if they satisfy a condition which is implied by C-convexity.

# 16.00-16.45 Jockum Aniansson

Fischer kernels and Cauchy problems for the wave equation.

Let P(W) and Q(Z) be two polynomials in n complex variables. We want to decompose the exponential kernel  $e^{W\cdot Z} = \exp(w_1z_1 + \cdots w_nz_n)$  into a sum  $e^{W\cdot Z} = h(W,Z) + g(W,Z)$ , where  $P(D_Z)h = 0$ , and Q(Z) divides  $g \cdot H$  there are unique entire functions h(W,Z) and g(W,Z), of exponential type  $w \cdot t \cdot Z$ , performing this, then we call them Fischer kernels. We will give some concrete examples.

One aim of the talk is to show how the solution  $g \cdot h$  to the inhomogeneous Cauchy.

One aim of the talk is to show how the solution  $\varphi$  to the inhomogeneous Cauchy-Goursat problem in  $\mathbb{R}^n$ ,

$$\begin{cases} P(D) \varphi = \psi \\ \varphi = f \text{ when } Q = 0, \end{cases}$$

can be linked the Fischer kernels

# 17.15-18.00 Nikolay Shcherbina

Hulls and Levi-flat surfaces.

A survey of the last results and a currant state in the theory of hulls and Levi-flat surfaces will be given.

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Sunday May 7.

9.00-9.45 Lars Filipsson

PDE-preserving polynomial interpolation

Kergin interpolation, a multivariate generalization of Lagrange interpolation, is PDE-preserving in the sense that for any homogeneous polynomial q,

 $q(D)f = 0 \implies q(D)Kf = 0,$ 

where Kf is any Kergin polynomial of f. It turns out that Kergin interpolation can be characterized in terms of this property. Indeed, if a continuous linear projector is such that it takes holomorphic functions into polynomials of degree d and furthermore interpolates function values at d+1 points, then it is a Kergin operator. (Joint with Jean-Paul Calvi)

10.00-10.45 Juhani Riihentaus

Subharmonic Functions: Non-Tangential and Tangential Boundary  $\operatorname{Be-havior}\nolimits.$ 

It is a classical result that every subharmonic function, defined and  $\mathcal{L}^p$ -integrable for some p>1 on the unit disc is for almost all  $\theta$  of the form  $o((1-|z|)^{-\frac{1}{p}})$ , uniformly as  $z\to e^{i\theta}$  in any Stolz domain. Recently Stoll improved this result by showing that for domains in  $\mathbb{R}^n$ ,  $n\geq 2$ , with  $C^1$  boundary, every subharmonic function satisfying a certain weighted integrability condition has weighted non-tangential and tangential boundary limits. We improve Stoll's result still further by relaxing both the weighted integrability condition and also the assumption of  $C^1$  boundary of the domain. Especially, we extend the result for domains whose boundaries are of finite d-dimensional Hausdorff measure (i.e. d-sets) and which in addition satisfy a rather mild accessibility condition.

11.00-11.45 Hasse Carlsson

Harmonic analysis and several complex variables

I will survey some aspects of the interplay betwee harmonic analysis and several complex variables. In particular I will discuss how the so called T1-theorem (for homogeneous spaces) about  $L^2$ -boundedness of singular integrals can be used to prove the duality between  $H^1$  and BMOA and atomic decomposition of  $H^1$ .

#### Preliminär deltagarlista Nordan2000, Örnsköldsvik

Göteborg Stockholm

Stockholm

	OlaO	aliM bileavo
	Sundsvall	Ahag, Per
	Wien	Zinner, Lucas
	Umeå	Xing, Yang
	Umeå	Wiklund, Jonas
(endast fredag)	. səmU	Wikström, Frank
	Amsterdam	Wiegerinck, Jan
ħ	Krasnoyarsk	Tsiikh, August
	Sundsvall	Sigurdsson, Ragnar
*	Göteborg	Sandberg, Sebastian
	Stockholm	Sadykov, Timur
	Stockholm	Rullgård, Hans
	Göteborg	Roginskaya, Maria
	Charkov	Rashkovskij, Alexander
	Santa Barbara	Putinar, Mihai
	öjxäV	Petersson, Henrik
	Stockholm	Passare, Mikael
	Umeå	Markström, Klas
	Göteborg	Lindholm, Niklas
	Amsterdam	Lemmers, Oscar
	Uppsala	Kutzschebauch, Frank
	Helsingfors	Koskenoja, Mika J.
	Uppsala	Khimshiashvili, Georg
	Stockholm	Juhlin, Robert
	Uppsala	Ivarsson, Björn
	Sundsvall	Fällström, Anders
	Gävle	Forsberg, Mikael
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	Stockholm	Ebenfeldt, Peter
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	Norrköping	Carlehed, Magnus
	Krasnoyarsk	Bushueva, Natlia
	Beer-Sheva	Brudny, Alexander
	Umeå	Backlund, Ulf
	Göteborg	Andersson, Mats
	Linköping	Alexandersson, Lars
		Deltagare
	Stockholm	Oktem, Ozan
	Grodeto	Shcherbina, Nikolay
	Joensun	Riihentaus, Juhani
	olsO	Løw, Erik
	Auckland	Levenberg, Norman
	Uppsala	Kiselman, Christer
	Lund	Hörmander, Lars
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	Göteborg	Carlsson, Hasse
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Øvrelid, Nils

Boman, Jan

Berndtsson, Bo

Föredragshållare Aniansson, Jochum